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## ERRATA

p. 485 , fn. 13: for "especiall page" read "especially pages"
p. 489: Figs. 4 and 5 were not printed at the correct size so as to have the same size of units, as described in the captions, i.e. 8 units across the width of the clavisimbalum should equal the units 1-9 in the clavicordium. (Measuring the units 19 instead of 0-8 corrects the error due to the binding: see p. 504)

Figs. 4 and 5 are reproduced below in the correct relationship (providing this document is printed at $100 \%$ on DIN A4 paper).

Fig. 4: Strohmayer's plan 1b. Clavisimbalum with units the same size as the clavicordium. © Wolfgang Strohmayer, 'Traditional design principles in the early history of keyboard instruments, part I, Clavichord International vol.6, no. 1 (May 2002), 15


Plan $1 b$, with the square, the letters $g, i, k, k^{\prime}$ and dividing mark enhancement added.

Fig. 5: Strohmayer's plan 2. Clavicordium with units the same size as the clavisimbalum. © Wolfgang Strohmayer, 'Traditional design principles in the early history of keyboard instruments, part I, Clavichord International vol.6, no. 1 (May 2002), 15


## Arnault's clavicordium: three solutions for a discrepancy

An interesting discrepancy arises from comparing Henri Arnault de Zwolle's instructions for designing the clavicordium with his drawing of the keyboard, which amounts to 3 mm in the manuscript, an error which is too large to be dismissed as an inaccuracy of draughtmanship (see fig. 1 below). This had been noticed by Le Cerf, ${ }^{1}$ but Wolfgang Strohmayer in a brilliant and thought-provoking analysis of Arnault's designs for the clavisimbalum and clavicordium, described hitherto unknown relationships between the instruments leading to an explanation for the discrepancy. ${ }^{2}$ Strohmayer inferred "Even in this example one becomes aware of the actual character of the text instructions: both of these passages read like a superficial description of an already existing drawing but are in no sense to be understood as exact instructions. As a result we must assume that Arnault for some reason or other reproduced the underlying Geometry in a 'simplified' fashion." ${ }^{3}$ That these two designs should not be seen as separate is one of Strohmayer's central theses: "[...] both drawings derive from the same geometrical idea ... there are the four letters ' $\mathrm{g}, \mathrm{i}, \mathrm{k}$, and h ' on the clavicordium keyboard which are simply the same graduations as on the ... clavisimbalum ... and are designated by exactly the same four letters [...]." ${ }^{4}$

1 Georges Le Cerf and Edmond René Labande: Les Traités d'Henri-Arnaut de Zwolle et de Divers Anonymes, Paris 1932 (reprint with comments by François Lesure, Internationale Gesellschaft für Musikwissenschaft, Documenta Musica, $2^{\text {nd }}$ series, IV, Kassel 1972), 18, note 1. Le Cerf and Labande give their French translation starting on page 17 alongside the Latin. Fol. 129 appears in facsimile as Plate IX, containing both drawings and text.
2 Wolfgang Strohmayer's first publication on this subject was Versuch einer Rekonstruktion zweier Instrumentenpläne des Arnault de Zwolle (um 1440), in: Österreichische Musikzeitschrift, no. 41, issue 1, ed. by Elisabeth Lafite and Marion Diederichs-Lafite, Vienna 1986, 2-6. Later publications deal with this theme in more detail and are cited separately below.
3 Wolfgang Strohmayer: Wertoolle Strukturen im Traktat des Arnault de Zwolle, in: Musicologica Austriaca, vol. 19, ed. by Monika Fink and Rainer Gstrein, Vienna 2000, 179: My translation, the original reads: "Schon an diesem einen Beispiel gewahrt man den eigentlichen Charakter der textlichen Anweisungen: Diese beiden Textstellen lesen sich wie eine oberflüchliche Beschreibung einer bereits existierenden Zeichnung und sind keineswegs als exakte Konstruktionsanweisung aufzufassen. Wir müssen demzufolge annehmen, daß Arnault aus irgendeinem Grund die zugrundeliegende Geometrie ,vereinfacht' wiedergegeben hat."
4 Wolfgang Strohmayer: Traditional design principles in the early history of keyboard instruments, parts I \& II, in: Clavichord International, vol. 6, no. 1, ed. by Koen Vermeji, Bennebroek (May) 2002, 15-16, I translated this article into English. I have used upper case letters for these designations in this article for greater visual clarity.

As a result of his investigations he concluded that the two instruments were related by a scale ratio of $66 / 7: 8(93 / 7: 8)$ and both were designed so as to exemplify the lengths $15 / 7,39 / 7$, and $66 / 7$, the latter value being were derived from the practice of Gothic architects in constructing an arch window. Thus, the keyboards of the clavisimbalum and clavicordium, which have the same width, conform to this scale ratio; thus, the clavicordium's keyboard was not completed following the instructions in the Latin text.

Strohmayer's hypothesis was elaborate, but a simpler explanation of the discrepancy of text and drawing might be that an error was made in drawing the clavicordi$u m$ 's keyboard, the width of which was then used to design the clavisimbalum. The text can easily be modified to make the instructions clear and unambiguous.

This essay clarifies the instructions for the drawing and investigates the implications of the instructions for the size of the keyboard. The second part examines Strohmayer's interpretation of the supposed underlying structures in Arnault's two drawings. In the third part a further solution is suggested to account for the discrepancy between the instructions and the clavicordium's keyboard. It is this third analysis, finding a scale ratio of 7:6 between the drawings and a keyboard width $2 / 3$ of the case length, which is advanced as the correct solution for the discrepancy.

## Part I: The keyboard size in Arnault's clavicordium

Arnault's clavicordium appears on fol. 129 with the fimbrium (string division) at the top, the clavichord in the middle, an organ pipe below this, and text under the pipe. ${ }^{5}$ The reader can refer to fig. 1 below for visual orientation regarding the clavicordium. Here follows a partial translation, which gives the instructions for the keyboard size. I have kept the translation close to the original in content and voice, even when this results in inelegancies, since these are in the original, which is written in a shorthand style. My explanatory comments are given in square brackets:

[^0]Le Cerf, Labande, Les Traités d'Henri-Arnaut de Zwolle, 1972, fol. 129 appears in facsimile as Plate IX. Le Cerf and Labande give their French translation on page 17-18 alongside their transcription of the Latin; the Latin text translated here starts on line 7 of fol. 129v, in facsimile as Plate $X$.

## Commentary on the text

The text concerning the instructions on the keyboard will become clearer if broken down into the separate steps:

Step 1. The distance between the keys $f$ and $f^{2}$ at the tangents is transferred to the front of the keyboard by dropping two perpendiculars. The distance between these two lines is called GH. These perpendicular lines are only intermediate steps required for finding the edges of the $f$ and $f^{2}$ keyheads, and are not required later. However, they provide a possible source of confusion, as is described below.

Arnault does not make this detail of working clear to the reader, and this bears comparison with some other descriptions which are also far from clear. This might be explained by the shorthand style, but it is also possible that he was quoting sources which were not complete, or which he had not fully understood. ${ }^{6}$

Step 2. The distance GH is divided into 14 parts.

Step 3. The right side of the f key (seen from the player) is half a part towards H from the perpendicular line $G$. Letter $G$ is therefore in the middle of the $f$ keyhead.

Step 4. The right edge of the f key is labelled with the letter I. Although not stated in the description that $I$ is the right edge of the $f$ keyhead, $I$ is labelled on the drawing.

Step 5. We then take half a part again (as in step 3) but now from H: H marks the line dropped perpendicularly from the $\mathrm{f}^{2}$ tangent. Here is the vagueness: in which direction? Do we take the same direction as in step 3, from G to H (i.e. towards the treble)? If we go from $H$ towards the treble then we will have the right edge of $f^{2}$ half a part on the treble side of H . This is not what is drawn. What we find on the drawing is that at the place H should be we have K written in tight against the line, just as I is drawn close to the division between the $f$ and $g$ keys. Furthermore, H is written in the middle of the $f^{2}$ keyhead, in an analogous fashion to $G$ being in the middle of the $f$ keyhead. As a result we find that $I$ and $K$ as drawn are $131 / 2$ divisions apart. This discrepancy was noticed by Le Cerf and Labande, who described H as being a half part too far to the right. ${ }^{7}$ This is also the discrepancy which Strohmayer refers.

[^1]Since the text describes the distance IK as containing 13 divisions, a resolution of this vagueness is simple: since it is stated there are 13 natural keyheads between the $f$ and $\mathrm{f}^{2}$ keys, we may infer that what is intended for step 5 is that in taking the half part from H we should put it to the left of H , in a mirror image fashion of step 3.

Thus the instructions could be corrected and completed thus:

Step 5. The left side of the $f^{2}$ key is half a part towards $G$ (i.e. towards the bass) from the perpendicular line $H$. (Letter $H$ is now in the middle of the $f^{2}$ natural keyhead.)

Step 6. Label the left edge of the $\mathrm{f}^{2}$ key with the letter K.

In this way we would have the distance GH equal to 14 parts, I is half a space towards the treble from G, K would be half a space towards the bass from H, IK would mark the space for 13 natural keyheads between $f$ and $f^{2}$ and be equal to 13 units of the 14 units designated by GH. This interpretation eliminates the vagueness of the text and brings the number of keys and the units apportioned to them into agreement.

The montage fig. 1 (with the correct keyboard in front of the clavicordium) shows (approximately) how the keyboard would have been drawn if the instructions had been followed with the middle of the $f^{2}$ keyhead on the line of the $f^{2}$ tangent. ${ }^{8}$

## The discrepancy between the instructions and the drawing

I have resolved the text so that it yields an unambiguous and self-consistent set of instructions how to lay out the keyboard, but Arnault's keyboard drawing does not have the width given by Arnault's instructions as I have completed them. Given the standard of accuracy otherwise found in the drawing, the error (about 3 mm ) would be surprisingly large and therefore probably too large to be explained by a slip of the pen. Can this be explained?

It is apparent that the keyboard drawing has been started using the instructions since $G$ and I are in the correct place, that is, in accordance with the text. The letter H is

[^2]

Fig.1: A montage showing the discrepancy between the clavicordium drawing and a keyboard (the lower one) designed following the instructions. Montage © Denzil Wraight
also found in the middle of a natural key, in the same way as step 3 locates $G$ in the middle of the $f$ keyhead. The step missing for the [completed] instructions and the drawing to be in exact agreement is step 5 .

What we find on the drawing is consistent with the following operations:
a) step 5 having been omitted (The left side of the $f^{2}$ key is half a part towards G from the perpendicular line H , i.e. the reverse direction of step 3, which found the position I.)
b) step 6, the labelling of the left edge of the $f^{2}$ keyhead, having been carried out, but with K being placed next to a line drawn through the temporary perpendicular line H of step 1 .

Thus, a simple explanation is that step 5 has simply been overlooked at the stage of drawing the instrument and the perpendicular from $\mathrm{f}^{2}$ was confused with the correct position for k .

## The size of the keyboard

Since Arnault tells us on fol. 129r how the organ pipe lengths or the string lengths of a clavichord may be found from the fimbrium, and the drawing shows that the fimbrium illustrated corresponds with reasonable accuracy to the clavichord, we may conclude that the fimbrium is the basis of the string lengths for this clavichord design. This is a basic feature of the design to which Huber has drawn attention. ${ }^{9}$

In the design itself, as Arnault writes, the longest string for $h$, (or $b$ in the English notation) is to be placed between the $1^{\text {st }}$ and $12^{\text {th }}$ divisions marked on the long side of the case, i.e. it is 11 units long. ${ }^{10}$

This being so, the instructions for making the fimbrium enable us to reconstruct the correct length, in modules, for the f and $\mathrm{f}^{2}$ strings and thereby determine the keyboard size. The fimbrium instructions start from the length MN and find $\mathrm{f}^{\#}$ as the first interval (at $2 / 3$ of MN ), then proceed by way of $\mathrm{c}^{\#}$ and a circle of fifths ending at h (b in the English notation). ${ }^{11}$ Thus, if one follows a chain of six fifths from $h(=11$ units) to $\mathrm{f}^{3}$ one can calculate the exact length for f , which is 7.7257 units (rounded to 4 decimal places). It follows that $\mathrm{f}^{2}$ must be a quarter of this, that is 1.9314 units, and that the distance $\mathrm{f}-\mathrm{f}^{2}$ is 5.7942 units.

Since $\mathrm{f}-\mathrm{f}^{2}=\mathrm{GH}$, and 14 keywidths, we can calculate that $13 / 14$ of this gives us the 13 keys between I and K on the drawing, i.e. 5.3804 units. Consequently the width of the whole keyboard is $22 / 13$ times this figure, or 9.1052 units, if the instructions are implemented correctly. Thus, it is clear that the keyboard width is not a whole number of units. Furthermore, the keyboard width, expressed in units, is so inconvenient that it cannot easily be taken as the starting point for the clavichord design.

## The size of the keyboard as drawn

There is a clear discrepancy between the drawing and the instructions as I have completed them. Since K occupies the place of H , and H should be the middle of the keyhead for $\mathrm{f}^{2}$, $\mathrm{K}^{\prime}$ s position is half a part to the right of where it should be. The distance IK as drawn is then actually $13^{1 / 2} 2$ divisions of the correct GH, which can be calculated as 5.5373 units. The total keyboard width as drawn is theoretically 9.4554 units, not the 9.1052 units given by the correct implementation of the instructions.

## The significance of the length IK

Why should there be such a complicated procedure for finding the length IK? The instructions could have been given as: make the keyboard width $91 / 2$ parts (case =

10 There is a small difference (a Pythagorean comma) between the length MN and the longest string h which results from the procedure of using a circle of fifths to derive the lengths. The string length $h$ is therefore correctly shown on the fimbrium as being slightly shorter than MN, as Le Cerf explained, page 15 , note 1 .
11 The instructions are given on fol. 129 r , Plate IX in Le Cerf and Labande, under the drawing of the organ pipe.

14 parts) and have the first key in the bass start at the beginning of the $3^{\text {rd }}$ part. This would have resulted in a keyboard of comparable size to what we have, positioned in almost the same place in the case.

The answer, I believe, lies in an actual manufacturing practice which is revealed with these instructions, although it is not explained. Most of the keylevers of any fretted clavichord of this size will have to be "cranked" towards the left or right so as to place the tangent at the correct position relative to the string. However, the instructions call for two keylevers to be straight, the $f$ and $f^{2}$; all the other levers are cranked by some amount. The clavichord no. 2 in Leipzig is an extant instrument which shows almost exactly this scheme: the straightest levers are at $f$ and $\mathrm{g}^{2.12}$ When one considers that this clavichord uses a keyboard size smaller than in Arnault's instrument, it is clear that the $\mathrm{g}^{2}$ of no. 2 is actually close to the $\mathrm{f}^{2}$ position of the design reported in Arnault's manuscript.

Obviously the best distribution of cranking is an empirical matter and the possible solutions are limited by practical considerations, but it is interesting to note that the $f$ notes are chosen in this design. This procedure of basing a scale design on the $f$ notes is widely found in $16^{\text {th }}$-century Italian virginals and harpsichords, which may be the continuation of an older tradition. ${ }^{13}$

Thus the instructions, according to my interpretation, are far from being a "superficial description" as Strohmayer saw it, but probably reflect actual manufacturing practices. John Koster has drawn attention to the clavicordium's string to case length ratio of 11:14, which he also finds exemplified in a virginal by Jos Karest of $1548 .{ }^{14}$ Herbert Heyde has observed that the ratio of the case width to case length in two extant clavichords is $3: 14$, just as in the Arnault clavicordium. ${ }^{15}$ Thus, there are three

12 This can be clearly seen in the drawing published by Edwin Ripin: The Early Clavichord, in: The Musical Quarterly, vol. 53, issue 4, ed. by Paul Henry Lang, Oxford (October) 1967, 518-538; Leipzig no. 2 is on page 529 , labelled C. One could consider the position for the straight lever in the bass to have been set between e and f since both are slightly cranked, but in opposite directions.
13 Denzil Wraight: The stringing of Italian keyboard instruments c. $1500-c .1650$, (Ph.D. diss., Queen's University of Belfast), Belfast 1997, part I, especiall page 121-123.
14 John Koster: Toward the Reconstruction of the Ruckers' Geometrical Methods, in: Kielinstrumente aus der Werkstatt Ruckers - zu Konzeption, Bauweise und Ravalement sowie Restaurierung und Konservierung, Bericht über die internationale Konferenz vom 13-15 September 1996 im Händel-Haus Halle, ed. by Christiane Rieche, Halle an der Saale 1998, 30-31.
15 Herbert Heyde: Musikinstrumentenbau, Wiesbaden 1986, 146-147. These are the clavichords Leipzig no. 2 and Dominicus Pisaurensis 1543, both in the Museum für Musikinstrumente der Universität Leipzig.
significant details suggesting the clavicordium belongs in a European tradition of instrument making. Arnault may not have been the originator of the instructions.

## Part II: An examination of Strohmayer's analysis

In his autobiography the great French mathematician Paul Lévy wrote that in order to interest children in geometry, one should proceed as quickly as possible to theorems they are not tempted to consider evident. ${ }^{16}$ One fascination of Strohmayer's hypothesis is that some of his theses are not at all evident; that he finds correspondences in the two drawings which hitherto had not been noticed. Those which I have found surprising are the following:

1. That both drawings should be labelled GI and IK to refer to similar lengths.
2. That GI and IK should apparently have the same size (expressed in their respective units) in both instruments.
3. That the length IK should apparently be an essential structural part of the clavisimbalum.
4. That a geometrical construction (a right-angled triangle with sides 8 and 5) for the two scales should exactly equal a numerical scale factor ( $66 / 7: 8$ ).
5. That the lengths GI and IK in the reconstruction should have exactly the sizes 15/7 and 39/7 (respectively).
6. That IK should have a particular significance making it worthy of incorporation in a design.

## 1. That both drawings should be labelled GI and IK to refer to similar lengths

Firstly, it should be noted that the length GI in Strohmayer's interpretation of the clavicordium places $G$ at the bass end of the keyboard. As a result GI contains the width of 5 natural keyheads. This is different from the manuscript clavicordium drawing where $G$ is placed (and described in the text as being) in the middle of the $f$ keyhead. Thus, GI in the clavicordium drawing is only half a key natural's width. This new
definition in Strohmayer's reconstruction of GI cannot be faulted since he is creating the rules for the structure, which he supposes to underlie the "simplified" drawing of the manuscript. However, only through the artificial definition is it possible to find the relationships of GI and IK in both instruments, so this new definition underpins the entire argument. Strohmayer shows both the original position of $G$ and his newly-defined position in fig. 2. ${ }^{17}$ Whether his definition is justified depends on the explanatory power of his entire hypothesis.


Fig. 2: Strohmayer's re-interpretation of the clavicordium's labels (below the keyboard). © Wolfgang Strohmayer: Traditional design principles in the early history of keyboard instruments, part I, Clavichord International, vol. 6, no. 1 (May 2002), 16

Although the use of the labels G, H, I, and K in both instruments led him to consider that they might stand for the same idea in both instruments, the new definition of the position G resulted in GH being widely different, and brought them even further apart. So while Strohmayer wishes to find a similarity of purpose in the labelling which affects GI and IK he ignores the consequences of his new definition which do not fit the hypothesis. This emphasises the artificiality of the new definition of GI.

## 2. That GI and IK should apparently have the same size (expressed in their respective units) in both instruments

Strohmayer shows how the lengths GI and IK are to be found in both instruments in fig. $3 .{ }^{18}$

17 Source: Strohmayer, Traditional design, 2002, part I, fig. 2.
18 Source: Strohmayer, Traditional design, 2002, part I, fig. 1 and 2.


Fig. 3: Strohmayer's interpretation of the common elements GI and IK with a new definition of GI in the clavicordium (label below the keyboard). © Wolfgang Strohmayer: Traditional design principles in the early history of keyboard instruments, part I, Clavichord International vol. 6, no. 1 (May 2002), 16

Then with the aid of a drawing of the clavisimbalum (plan 1b), which is reduced to have its units the same size as those of the clavicordium (plan 2), he claims that these lengths GI and IK are then physically the same. ${ }^{19}$

A special gauge was produced with which the reader could compare the dimensions and according to this scale GI and IK appear to be identical in both instruments. Thus Strohmayer claims: "Here we are concerned with clear and measurable correspondences which are beyond any objection or doubt." ${ }^{20}$ Furthermore he claims that: "This means that the same distances gi and ik in the reduced clavisimbalum drawing (plan 1 b ), contrary to appearances, are also defined by whole number division!", by which he means $15 / 7$ and $39 / 7$ respectively. ${ }^{21}$ Although Strohmayer stops short of claiming that the (scaled) lengths GI and IK are identical in both instruments, he encourages this impression to arise with his two drawings brought to the same scale.

In fact, if the correct manuscript size for both drawings is used and the clavicordium's IK length is scaled up by the scale factor ratio $(\sqrt{ } 89) / 8$ (to be discussed in section 3 below), then the clavisimbalum's IK is some 1.5 mm less than the required length but

19 Source: Strohmayer, Traditional design, 2002, part I, plan 1b, 15.
20 Strohmayer, Traditional design, 2002, part I, 16.
21 Strohmayer, Traditional design, 2002, part I, 16.

Fig. 4: Strohmayer's plan 1b. Clavisimbalum with units the same size as the clavicordium. © Wolfgang Strohmayer: Traditional design principles in the early history of keyboard instruments, part I, Clavichord International vol. 6, no. 1 (May 2002), 15


Plan 16 , with the squace, the lenees $g$, $L k$, $k^{\prime}$ and dividing mark athinnament added.


Fig. 5: Strohmayer's plan 2. Clavicordium with units the same size as the clavisimbalum. © Wolfgang Strohmayer: Traditional design principles in the early history of keyboard instruments, part I, Clavichord International vol. 6, no. 1 (May 2002), 15
this is not easy for the reader to see since Strohmayer's drawings have been reduced in size. ${ }^{22}$ The gauge provided even shows GI to be 1 mm too long on the clavisimbal$u m$ because of a slight error in the position of the enhancing mark for I.

It would not benefit Strohmayer's argument to suggest that some inaccuracy of drawing should be allowed for since the task Arnault was faced with was not to make a reduced-scale clavisimbalum fit a clavicordium; it was to create lengths for GI and IK in each drawing which have the same unit length, albeit with different physical lengths. This requires that an appropriate construction be used in either drawing.

We obtain an interesting insight into the matter if we recall that Arnault's instructions for constructing the clavisimbalum are geometrical: even if a geometrical operation is imperfectly executed, the underlying idea is exact and can therefore be calculated. It is thus relatively easy to calculate the lengths for GI and IK in the clavisimbalum, which are as follows. (The values in brackets are those which Strohmayer's hypothesis require).

$$
\begin{array}{lll}
\mathrm{GI}=2.0397 & \text { Arnault's GI is }-4.8 \% & (15 / 7=2.1429) \\
\mathrm{IK}=5.5347 & \text { Arnault's IK is }-0.66 \% & (39 / 7=5.5714)
\end{array}
$$

From this analysis it is obvious that GI and IK do not have the size in Arnault's construction for the clavisimbalum which Strohmayer's hypothesis requires, even though they appear to be similar in the reduced-scale drawings. In fact, the percentage error for GI $(=4.8 \%)$ is greater than the discrepancy originally noted by Strohmayer in the keyboard ( $=3.55 \%$ ). Of course, the absolute magnitude of error is smaller for GI, and therefore less obvious, since only about 2 units are involved, which is why Strohmayer's claim of "clear and measurable correspondences" for GI and IK is initially plausible.

The issue with the clavicordium drawing is less clear. Strohmayer observes a keyboard width in the manuscript which he declares to be $93 / 7$ (= 9.4286) units; this is a crucial point in his argument, which is derived initially from a correspondence between 3 units of the case and 7 natural widths. This is correct if we compare h-a

[^3]with 3 units, but not if we compare $c^{2}-h^{2}$, which are marginally narrower. ${ }^{23}$ The fact that the two keyboards have evidently the same width is an important detail, but it is the supposed size of $93 / 7$ units which creates the special relationship that Strohmayer believes underpins the two drawings.

If we calculate the width of the keyboard using the 14 units of the case length (= 198.5 mm ) and the physical length of the keyboard ( $=133.5 \mathrm{~mm}$ ), then we find it to be 9.4156 units; Strohmayer's 9.4286 is only $0.14 \%$ too large, which is remarkably close. Whether this keyboard size was intended to be $93 / 7$ units, or was constructed in some other way is the issue. If one could answer this question simply then there would be no need for the length of this article.

Of course, GI and IK in the clavicordium (of fig. 2) are exactly 15/7 and 39/7 units respectively because Strohmayer has defined them to be so through a keyboard width which he gives as 66/7 (=93/7). However, we have seen that GI and IK do not have exactly the size in the manuscript drawing of the clavisimbalum which Strohmayer's hypothetical "underlying structure" requires.

Given the lengths Strohmayer went to in order to analyse the size of IK in his re-construction of the clavisimbalum, it would have been possible for him to have revealed that GI and IK do not have the size his hypothesis requires. Strohmayer uses the approximately correct GI and IK lengths of the drawings to suggest what he considers to be the underlying intention of creating identical GI and IK dimensions. His reconstruction then implements what he considers to be the underlying structure, which is exact. Thus, the lengths GI and IK in Arnault's clavisimbalum drawing are not constructed exactly as $15 / 7$ or $39 / 7$, although this fact does not refute Strohmayer's hypothesis, since his argument is about the "underlying structure".

## 3. That the length IK should apparently be an essential structural part of the clavisimbalum

Strohmayer's second presentation is to relate the sizes of the instruments through the 8 unit module, which is used to construct the clavisimbalum.

23 In making this comparison one should avoid the first unit, which is artificially short. I will return to this slight discrepancy in part III.


Fig. 6: Strohmayer's fig. 3-5. © Wolfgang Strohmayer: Traditional design principles in the early history of keyboard instruments, part I, Clavichord International vol. 6, no. 1 (May 2002), 16

From his presentation it appears as if the length IK is an essential part of the design which has hitherto not been recognised. In Strohmayer's fig. 3, the mysterious length IK is shown on the clavisimbalum drawing which apparently leads to a triangle in his fig. 4 with the right end of the hypotenuse labelled as K. In his fig. 5 the hypotenuse of the $8-5-\sqrt{89}$ triangle is then used to find the size of the units for the clavisimbalum. ${ }^{24}$ The impression is thereby created that IK is necessarily connected with the 8-5- $\sqrt{89}$ triangle, through which the clavisimbalum and clavicordium sizes of units are related. ${ }^{25}$

This is incorrect, and Strohmayer's figs. 19, 20 and 21 show the situation correctly, even though they are of such size that this detail might escape the reader's attention. ${ }^{26}$ The fact is that the line IK, in either Arnault's drawing or in Strohmayer's

24 The description of the hypotenuse as $\sqrt{ } 89$ is mine and was not used by Strohmayer in the text, although in his notes 10 and 11; the squares of the lengths of the triangle are 64,25 and 89 . Although the expression $\sqrt{ } 89$ for the length of the hypotenuse is anachronistic it serves to show that the length could only have been developed geometrically at this time, not as a whole number.
25 This error is perpetuated in Strohmayer, Mathematische Gestaltungsprinzipien in den Instrumentenplänen des Henri Arnault de Zwolle, in: Das Österreichische Cembalo, ed. by Alfons Huber, Tutzing 2001, 35-77, fig. 5-7, 30, and 40.
26 Wolfgang Strohmayer, Traditional design principles in the early history of keyboard instruments, part III, in: Clavichord International, vol. 6, no. 2, ed. by Koen Vermeij, Bennebroek (November) 2002, 44-46.
interpretation, when extended towards the lefthand vertical axis CD, does not pass through the mark for the end of the $8^{\text {th }}$ unit (labelled $K^{\prime}$ on Strohmayer's plan 1 b ), which creates the square with 8 unit sides. ${ }^{27}$ The impression created that IK belongs to the $8-5-\sqrt{ } 89$ triangle is misleading.

## 4. That a geometrical construction (a right-angled triangle with sides 8 and 5) for the two scales should exactly equal a numerical scale factor (66/7:8)

A tension which appears in Strohmayer's argument concerns the scaling factor for the two instruments. When the keyboard sizes are compared, the ratio of the units (clavicordium to clavisimbalum) is $93 / 7$ (or 9.429 ):8, or $66 / 7: 8$, but the geometrical derivation through the triangle with sides 8 and 5 units long results in a $\sqrt{89}$ (9.434): 8 ratio. The difference is extremely small, but the interpretations are not numerically identical. Strohmayer was satisfied that 66/7 represented a very good approximation (for the geometrically derived $\sqrt{ } 89$ ), which was therefore not in itself a problem. ${ }^{28}$ The similarity of geometrical and arithmetical approaches is part of Strohmayer's more general argument regarding the "fundamental compatibility of Geometry and Arithmetic" which he saw as motivating the Gothic architect, but which in practice involved approximations. ${ }^{29}$

## 5. That the lengths GI and IK in the reconstruction should have exactly the sizes 15/7 and 39/7 (respectively)

In Strohmayer's English language publication it is not clear why the lengths GI and IK in his reconstruction should have exactly the lengths $15 / 7$ and $39 / 7$ (respectively, see my fig. 3 above), but the matter is dealt with at more length in a later German language article. This shows how the triangle with sides $7-8-\sqrt{ } 15$ was developed so as to determine the point I and simultaneously yield exactly $15 / 7$ for the length GI. ${ }^{30}$ (Labels have been added for C, I and G so that the reader can see how this element fits in my fig. 3 above).

28 Strohmayer, Traditional design, 2002, part I, note 11.
29 Strohmayer, Mathematische Gestaltungsprinzipien, 2001, 57.
30 Strohmayer, Mathematische Gestaltungsprinzipien, 2001, 56-57.


Fig. 7: Strohmayer's Zeichnung 20 (with added labels for C, I and G). © Wolfgang Strohmayer: Mathematische Gestaltungsprinzipien in den Instrumentenplänen des Henri Arnault de Zwolle, in: Das Österreichische Cembalo, ed. by Alfons Huber, Tutzing 2001, 57

We see that, as a matter of fact, the length IK is not constructed in Strohmayer's version, just as it is not constructed in Arnault's. It is the result of defining I and K in separate operations. ${ }^{31}$ This shows that the length IK has no function in Arnault's drawing. In Strohmayer's it is only present in order to satisfy his hypothesis that a "single geometrical idea" is the basis of the two designs.

Furthermore, the length IK in the re-construction is not exactly correct, but this is not of great moment; it is the calculations Strohmayer adduces to define the length which are so tortuous that one has to wonder whether Arnault would ever have performed them in order to determine the length of IK. ${ }^{32}$

This leads to the only issue which I wish to consider more closely here. If the length IK is not even constructed, but merely the barely-calculable result of another operation, are we justified as seeing IK as a fundamentally important length in the basic design? For Strohmayer this does not appear to constitute a difficulty, who is satisfied that his $7-8-\sqrt{15}$ triangle design has achieved the length GI and the point I in one simple geometrical operation. In my view IK is inherently implausible as a central feature if the length IK is not known through construction.

31 In Strohmayer's later article, Mathematische Gestaltungsprinzipien, 2001, 63, he writes that the length ik is to be understood as "geometrically produced", where the exact phrase is "... die Strecke ik insgesamt als geometrisch verursacht aufzufassen ist."
32 Strohmayer, Mathematische Gestaltungsprinzipien, 2001, 62.

It would be possible to have used a much simpler, and exact, geometrical method to achieve the goal of $\mathrm{GI}=15 / 7$ and $\mathrm{IK}=39 / 7$ : this would be to have set the compass to $15 / 7$ and then to have scribed an arc from Arnault's G, then setting the compasses to $39 / 7$ and scribing an arc from K . Where the arcs cross (inside the 8 -sided square) one would find the intersecting point I. This method is so simple that I wonder why a more complicated method is needed. Was Strohmayer impelled by the logic of his own premise that Arnault's drawing was "simplified" and therefore had to find something more complicated, namely the construction of fig. 18 and 19? Strohmayer's guiding idea appears to be that the source could be found in the geometrical constructions of the Gothic architect. This appears to be the line of thinking illustrated by Strohmayer's Zeichnung 21 (fig. 8 below). ${ }^{33}$


Fig. 8: Strohmayer's Zeichnung 21 (with added labels for C, I and G). © Wolfgang Strohmayer: Mathematische Gestaltungsprinzipien in den Instrumentenplänen des Henri Arnault de Zwolle, in: Das Österreichische Cembalo, ed. by Alfons Huber, Tutzing 2001, 57

The right hand drawing shows the afore-mentioned right-angled triangle, with two sides of 7 and 8 , which is constructed geometrically in a half circle within the $8 \times 8$ unit square; the third side will be $\sqrt{ } 15$ units long. ${ }^{34}$ If the reader imagines this rotated clockwise through $90^{\circ}$, the bottom lefthand corner corresponds to C on the clavisimbalum and the $90^{\circ}$ angle in the triangle corresponds to I. The left hand drawing shows how the Gothic arch construction yields a point where the two arcs with radius 4 cross. The juxtaposition with the right hand drawing could suggest that they are the same, and this correspondence is developed more clearly in a later drawing (fig. 9, Zeichnung 32) with only these two drawings, so that it appears as if we have the

33 Source: Strohmayer, Mathematische Gestaltungsprinzipien, 2001, 57.
34 Strohmayer, Mathematische Gestaltungsprinzipien, 2001, 57. One should imagine that the horizontal line in this drawing is the first 8 units of the line CD of the clavisimbalum.
development of his right-angled triangle with two sides of 7 and 8 from the Gothic arch. ${ }^{35}$ No explanation is offered how we should understand this juxtaposition.


Fig. 9: Strohmayer's Zeichnung 32 (with added labels for C and G). © Wolfgang Strohmayer: Mathematische Gestaltungsprinzipien in den Instrumentenplänen des Henri Arnault de Zwolle, in: Das Österreichische Cembalo, ed. by Alfons Huber, Tutzing 2001, 63

In fact, in the left drawing, the point where the two arcs with radius 4 cross (drawn in bold), which appears to correspond to " I " in the right drawing, is not identical with it. The diagonal line in the right drawing to " I " is 7 units, but in the arch construction it is $\sqrt{ } 48$ or 6.9282 . Thus, the arch construction cannot be an exact construction for the triangle.

What Strohmayer perhaps means to suggest is that the arch "inspired" the triangle, or that within the normal range of approximation they were considered equivalent. This is an example of his two-layered method of presentation. There is, at one layer, a clear argument regarding sizes of constructions (e.g. the 7-8- $\sqrt{15}$ triangle). At another layer there is no specific claim made, merely a juxtaposition of visually similar elements; the reader is invited to draw his own conclusions. In the circumstances, and the absence of explanation, he is likely to be led by the juxtaposition to draw the incorrect conclusion that the elements are identical. In this way Strohmayer suggests connections between elements of Gothic architecture and his underlying structure for Arnault's design without actually claiming them, but there are also a bewildering number of juxtapositions which may leave the reader unsure what is being argued and what merely suggested. ${ }^{36}$

36 See for example the development of the clavicordium and clavisimbalum from the Gothic arch window in Strohmayer, Traditional design, 2002, part III, fig. 15, 16, and 21. The different scale sizes, and how these would have been implemented in the re-construction, are not clearly enough described that

We see that Strohmayer prefers a solution for the position of the point I based upon a historical approach to constructional design, even if it implies that Arnault might never have known how long IK was, rather than a mathematical treatment which exactly satisfies the requirements of the hypothesis.

## 6. That IK should have a particular significance making it worthy of incorporation in a design

What is significant for Strohmayer's general argument is that numerical relationships should be found between different systems. Thus, he draws attention to the 66/7 length as an approximation for the half circumference of a circle with a radius of 3 units (the Gothic arch window) and the width of the clavicordium with 22 naturals which has, according to his reading of the drawing, an actual width of $66 / 7$ units. In this way Strohmayer establishes a symbolic connection between the Gothic arch window and the width of the clavicordium's keyboard.

There is though no deeper significance for the values $\mathrm{GI}=15 / 7$ and $\mathrm{IK}=39 / 7$, which are the lengths he believed to be contained in both the clavisimbalum and clavicordium drawings. The only significance for $39 / 7$ (= IK) is to be found in the instructions for creating the keyboard, which Strohmayer rejected at the outset. The length IK has no function in Strohmayer's re-construction of the cases of the instruments. ${ }^{37}$

## A summary of Strohmayer's hypothesis

Although Strohmayer encourages the reader to believe that the lengths GI and IK have the same unit size in each drawing, it is not necessary for his argument that they be so. Indeed, the whole scheme is that there is an "underlying structure", and as a detailed analysis can show, this structure is marginally different than the actual clavisimbalum drawing. Indeed, one would expect this, otherwise there would be no need for a deeper layer of structure to explain the drawing.

The length IK is not a special part of the 8-5- $\sqrt{ } 89$ triangle which yields the geometrical definition of the scale ratio between the clavicordium and clavisimbalum, despite appearances to the contrary in some figures. In fact the length IK is not constructed

[^4]in Arnault's drawing or Strohmayer's re-construction of the clavisimbalum, so it has no function in the drawing.

Even though there are also many other elements which I have not described, Strohmayer's highly ingenious hypothesis of an underlying structure necessarily depends upon two details:

1. The re-defined length GI in the clavicordium; without this new definition there is no length in the clavicordium approximately corresponding to GI in the clavisimbalum. 2. The width of the clavicordium keyboard being 66/7 ( $93 / 7$ units wide). As will be seen in part III, the keyboard has a different size, so that Stromayer's interpretation of $93 / 7$ units has no empirical confirmation from the drawing.

The re-definition of GI in the clavicordium and the lack of significance of IK are related issues which bear upon the plausibility of the hypothesis. These are artificial constructs as employed by Strohmayer. This lack of significance for the lengths GI and IK is ultimately, in my view, what makes Strohmayer's interpretation implausible: it is an explanation, the core of which is without a motivation.

## Part III: A third explanation

For some time it seemed to me that there were only two plausible solutions to the discrepancy between the drawing of the clavicordium keyboard and the instructions for generating it:

1. A flawed execution of the instructions giving $13^{1 ⁄ 2}$ units for IK (instead of 13 units, as described in part I).
2. Strohmayer's hypothesis having the clavicordium in a scale relationship to the clavisimbalum of 66/7:8.

I was reluctant to entertain the idea that a clumsy error could explain the discrepancy. When one reviews the work involved in Arnault's drawings, one has to recognise the considerable practical skill involved in making such fine lines on paper, which require not only a reliable technique but also a good ruling pen and suitable ink. Some of the lines on the fimbrium are separated by no more than about 1 mm indicating the fineness of the work Arnault executed. Thus, the idea of a clumsy error seems to be an explanation of last resort.

Both explanations given above (parts I and II) yield keyboard sizes which are not greatly different. As we have seen, assuming solution 1, the calculated size of the keyboard width as drawn is 9.4554 units. Strohmayer's assumed width of $93 / 7$ units is in decimal terms 9.4286. The difference is so small that it appeared unlikely one could use the drawing to decide which solution is correct since 0.0268 units is only 0.38 mm and thereby little more than the thickness of an ink line.

If one tackles this problem by considering the practical execution of the drawing's instructions we see that even if the line of the f tangent has been correctly used for the centre of the f key (point G ), but the line of the $\mathrm{f}^{2}$ tangent has been incorrectly used for the right edge of the $\mathrm{f}^{2}$ lever (point K ), there should be 13 evenly spaced keys for $\mathrm{g}-\mathrm{e}^{2}$. One would then expect this spacing to have been used to produce the remaining key sizes.

A sensitive method to compare the keyhead spacing is to make a photocopy on transparent foil, then turn the foil over so that the bass end of the keyboard is compared with the treble end. ${ }^{38}$ Apart from minor errors due to a few keyheads being drawn slightly slanted (e.g. at $\mathrm{d}^{2} / \mathrm{e}^{2}$ ), the comparison reveals that the middle of the keyboard, i.e. the junction of the $\mathrm{e}^{1}$ and $\mathrm{f}^{1}$ keyheads, is not central; the deviation is almost 1 mm , which, given the accuracy otherwise found on this drawing, is surprisingly large (fig. 10).


Fig. 10: A comparison of the clavicordium keyboard showing that the middle of the keyboard, $\mathrm{e}^{1} \mathrm{f}^{1}$, is not centrally placed, and that the spacing of $\mathrm{h}-\mathrm{e}^{1}$ does not match $\mathrm{f}^{1}-\mathrm{h}^{2}$. Montage © Denzil Wraight

38 To increase the sensitivity of this test I enlarged the drawing by $x 1.41$ before making the foil. The size of page chosen for this publication does not permit these comparisons to be printed across the page in the size I used.

Furthermore, when the spacing of the treble half $\left(\mathrm{f}^{1}-\mathrm{h}^{2}\right)$ is compared with itself in reversed fashion, an even spacing is evident (fig. 11).


Fig. 11: A comparison of the treble half of the clavicordium's keyboard showing even spacing. Montage © Denzil Wraight

Similarly, the spacing of the bass half of the keyboard (h-e ${ }^{1}$ ) is also even (fig. 12), but does not match the spacing of the treble, as we saw in fig. 10.


Fig. 12: A comparison of the bass half of the clavicordium's keyboard showing even spacing. Montage © Denzil Wraight

What we evidently have here is a keyboard which has been drawn in two stages, with a slightly different spacing for each half of the keyboard. This asymmetry and uneven spacing is alone sufficient to dispel the idea that the clavicordium keyboard is the result of a partial execution of the instructions, as described above; it has been constructed in some other way. Thus, the first solution I suggested is insufficient to explain all the observable facts.

A striking feature of the keyboard comparison I gave above, to show the asymmetry and uneven division of the clavicordium's keyboard, is that it does not depend on knowing the absolute size of the drawing. That is a further factor which can now be considered to determine the relationship between the size of the units for the clavicordium and clavisimbalum.

Neither Le Cerf's edition, nor the Lesure facsimile of his edition, reproduced the clavicordium and clavisimbalum at the original size, but some $2 \%$ larger. These details were only brought to light when Koster examined the original manuscript and published the correct length for MN on the fimbrium, above the clavicordium of fol. 129, as $157 \mathrm{~mm} .{ }^{39}$ Additional information from Koster showed that the clavisimbalum should have the length $\mathrm{CD}=215 \mathrm{~mm} .{ }^{40}$ Careful comparison of the online publication of the manuscript, Koster's data, and the facsimile edition of Le Cerf shows that the facsimile edition gives the correct relationship between the instruments on fol. 128, $129,129 \mathrm{v}$, and 130 , even though the absolute size is too large. ${ }^{41}$

Further measurements made by Koster indicate that 14 units on the clavicordium are 198.5 mm long on the manuscript. ${ }^{42}$ Another observation by Koster also shows that Arnault was considering the dulce melos design at the same time, since he measured 198.5 mm for the internal case width of this instrument on two drawings. ${ }^{43}$ Thus, Koster observed that the clavicordium length is not a unique feature, but one part of a planned size for several instruments.

Strohmayer suggested that the units of the two instruments were related by the relationship $\sqrt{ } 89: 8$ (derived geometrically for clavicordium : clavisimbalum), with the clavisimbalum case width of 8 units then determining the clavicordium's keyboard width. On this basis one can calculate that if 14 units of the clavicordium are 198.5 mm (the length Koster measured), then 12 units of the clavisimbalum should be 200.6 mm ; there should be a 2.1 mm difference. ${ }^{44}$ This is a relationship which can be tested

39 Koster, Toward the Reconstruction, 1998, 31.
40 Personal communication, 29.8.2000.
41 This is not prima facie obvious since the reproduction of Le Cerf's edition might have yielded slightly different sizes for each sheet. The manuscript can be inspected online at http://gallica.bnf. fr/ark:/12148/btv1b90725989/f134.image.r=latin\%20AND\%207295\%20.langFR which is a link to fol. 128. The entire document can be downloaded free as a PDF file, single page views are also available as JPEGs.
42 Communicated during the course of writing this article, which was of considerable assistance in clarifying the issue of size and is hereby gratefully acknowledged.
43 Personal communication, 20.12.2013, for the dulce melos depicted on fol. 129v and fol. 130.
44 The difference between the geometrical relationship $(\sqrt{ } 89: 8)$ and the arithmetical $(93 / 7: 8)$ is pertinent at this stage but amounts only to a 0.1 mm difference over 12 units of the clavisimbalum and does not
empirically, and because more units are involved than in the width of the keyboard, any difference will be more readily apparent.

As a result of Koster's confirmation of the absolute size of several sheets of the manuscript, we now know that the drawings in the facsimile editions can be directly compared. Therefore, it is possible to suggest that 14 units of the clavicordium were probably intended to be identical with 12 units of the clavisimbalum, i.e. without the 2.1 mm difference which Strohmayer's hypothesis implies.

How good a fit we could expect from the standard of Arnault's draughtsmanship if he were intending to draw the same length can be investigated by comparing the first four units of the line CD on the clavisimbalum with the line BA (at the cheek). Fig. 13 shows the clavisimbalum in the lower part, with the line $C D$ running horizontally; part of the rose is visible as an orientation. The cheek has been aligned above $C D$, but with BA overlapping the line CD; part of the bridge and bentside is visible. Of course, it requires some familiarity on the part of the reader with the drawing to be able to recognise the various parts in this juxtaposition.


Fig. 13: The line CD of the clavisimbalum compared with the line BA. Montage © Denzil Wraight

There are up to 0.5 mm differences at the unit marks in distances which should be nominally identical. Thus, a difference of 0.5 mm for lengths intended to be identical would not be surprising.

We can now examine the size of the case unit on the clavisimbalum drawing with the foil technique, which shows us a close fit (within 0.5 mm ) of the $0-14$ clavicordium
materially affect the issue discussed. The calculation is 12 units $\times \sqrt{ } 89: 8=14.151$ units. If 14 units $=$ $198.5 \mathrm{~mm}, 14.151$ units $=200.6 \mathrm{~mm}$.
units laid over the $0-12$ units of the clavisimbalum. ${ }^{45}$ Fig. 14 shows a foil photocopy of the clavicordium laid over the clavisimbalum so that the beginning of unit 7 on the clavicordium is aligned with the end of unit 6 on the clavisimbalum. It will be seen that unit 14 of the clavicordium lies directly over the line EF so that it is hardly visible. ${ }^{46}$


Fig. 14: Units 7-14 of the clavicordium correspond to units 6-12 of the clavisimbalum.
Montage © Denzil Wraight
The fit of 14 clavicordium units against 12 clavisimbalum units is well within the range one would expect. This closeness of fit is easily distinguishable from the 2.1 mm difference required by Strohmayer's hypothesis, based on his geometrically derived $\sqrt{ } 89: 8$ scale factor. ${ }^{47}$

Further insight can be gained by using the reversed foil technique on the 13 units of the clavisimbalum's case, $C D$. If $C$ (of the line $C D$ ) on the foil is held over $D$ on the drawing then there is a discrepancy of about 0.5 mm at the $6^{\text {th }}$ unit. Examination by this method shows that the uneven spacing occurs in the last unit (12-D), which is slightly short. If we align the $12^{\text {th }}$ unit, using the reversed foil technique, with the starting unit at C then we find an even division. We may therefore conclude that the line CD was marked out in two stages, C-12 was accurately even, then a slightly shorter 12-D was added. ${ }^{48}$

45 Comparison was made from the line CB to the line EF. EF appears to mark the correct unit division, not the apparent ink dot about 0.5 mm nearer C . On CD (and other lines) it is possible to see small black dots at each unit division where ink has filled the small hole made by a divider. Thus, a unit division is often comprised of the dot and a line ruled through the dot.
46 This comparison was chosen mainly because a reproduction of the whole clavisimbalum would undoubtedly be reduced in this publication, so this size gives the best visibility, but also because the unit $0-1$ of the clavicordium is shortened due to the folding over of the page in the binding. As a result a correct alignment of 0 on the clavicordium against $C$ on the clavisimbalum is not possible from the published plate.
47 Strohmayer's arithmetical scale factor 66/7:8 gives a result only 0.1 mm different. I use the two terms interchangeably for most purposes, as Strohmayer intended.
48 This procedure is not so strange when we recall that the clavicordium case must have been constructed

Investigating the clavicordium's case units by the same reversed foil method we find that the $7^{\text {th }}$ unit is also not central, there being about 0.5 mm difference as a result of uneven division. On closer examination it transpires that the problem lies in the first unit ( $0-1$ ), which is obviously shorter. This is due to the fact that the leaves of the manuscript have been bound into a book and the left edge is close to the binding. Although in Lesures's facsimile version this is not obvious, the online view shows this more clearly, with part of the case wall being missing. Thus, the edge of the page being pulled down into the binding has shortened the first unit, as seen by a camera or flatbed scanner. When the first unit is supplemented with one of the correct size, then one finds that 14 units of the clavicordium end at exactly the line EF on the clavisimbalum, the $12^{\text {th }}$ unit. This also cautions that any comparisons of the clavicordium case length should not be made starting at the beginning of the first unit of the modern editions. ${ }^{49}$

In view of this exact correspondence we are entitled to conclude a 14 to 12 unit identity for the clavicordium and clavisimbalum, which allows us to infer a scale relationship of 7:6 (= 1.6666 ${ }^{\text {r }}$ ) between the two instruments, not the 9 3/7:8 (= 1.1786) postulated by Strohmayer. ${ }^{50}$

That the two instruments should be related in some way has long been considered, this line of thinking apparently having been initiated by Peter Kukelka who argued that the string length HN of the fimbrium, which was 11 units when used in the clavicordium, also forms the basis of the clavisimbalum, when one measures from the bridge to a line in front of the soundboard where the strings are excited. ${ }^{51}$

It is apparent that both keyboards were intended to have the same width, the actual difference being only about 0.2 mm . It follows, therefore, that the clavicordium key-

[^5]board is theoretically $2 / 3$ of the clavicordium case length $(8 / 12$ units clavisimbalum $=$ $x / 14$ units clavicordium), or $91 / 3$ units wide, which is not compatible with the $93 / 7$ Strohmayer had interpreted. ${ }^{52}$ The relationship of the keyboard widths is therefore 9 1/3:8.

When the two drawings are compared it will be seen that there is an interesting error in the clavisimbalum. The end of the $4^{\text {th }}$ unit on the 8 -unit line $C B$, counting from $C$, is not centred, as one can show with the reversed foil technique. The length CB is also about 0.5 mm longer than the 8 units on the line CD , counting from C . The reason for this appears to be that Arnault's ruler slipped marginally as he drew the line CD: the lack of straightness is visible, as is the correct mark for the start of CB. This largely explains why the end of the $4^{\text {th }}$ unit is not central on the line CB.

This length of 0-4 on CB also corresponds to the h-e ${ }^{1}$ keys in the clavicordium (fig. 15, where the clavicordium keyboard has been superimposed above the CB scale), but is evidently too long for the distance $\mathrm{f}^{1}-\mathrm{h}^{2}$ (fig. 16).


Fig. 15: The clavicordium keyboard has been superimposed above the $C B$ scale: $e^{1}-f^{1}$ corresponds with the end of the $4^{\text {th }}$ unit. Montage © Denzil Wraight


Fig. 16: The clavicordium keyboard has been superimposed below the CB scale: $\mathrm{e}^{1}-\mathrm{f}^{1}$ does not match the end of the $4^{\text {th }}$ unit. Montage © Denzil Wraight

52 Herbert Heyde, Musikinstrumentenbau, 1986, 146, gave the width of the clavicordium keyboard as 2/3 of the case length, but did not compare the clavicordium with the clavisimbalum. His assessments of the dimensions were approximate since he gave the space either side of the clavicordium keyboard as $1 / 6$ of the case length, even though it is obviously asymmetrically placed in the case, the difference being some 3 mm on the drawing.

Thus, Arnault could have started his clavicordium keyboard drawing by taking the halfway mark from the clavisimbalum plan. ${ }^{53} \mathrm{He}$ would then have found that the spacing he used for one half did not match the other and would have adjusted his divider spacing accordingly, which presumes that he produced a trial version of the keyboard division on another piece of paper. In fact, Arnault might not even have been aware of the discrepancy if instead of dividers he used an isoceles triangle with 12 lines leading from the tip of the triangle to the base, which was divided into 11 parts. ${ }^{54}$ He would simply have brought each half of the keyboard into alignment with this aid and transferred the division from it.

There are now two details in the drawings which suggest (but fall short of proof) that the clavicordium keyboard was derived from the clavisimbalum: Firstly, the fact that the width of the clavisimbalum CB is slightly more than 8 units on the line CD , and that the clavicordium keyboard has exactly the width of CB . Secondly, that the asymmetry of the clavicordium keyboard matches the non-central $4^{\text {th }}$ unit on CB of the clavisimbalum.

Arnault apparently chose to orient the clavicordium keyboard so that the $\mathrm{e}^{2}-\mathrm{f}^{2}$ keyhead junction is on a line with the $f^{2}$ tangent, which requires that he already had his keyboard division complete. There is no other simple alignment I have been able to find which he might have used to locate the bass or treble end of the keyboard, whether measured in units of the clavicordium or clavisimbalum. This fact, the asymmetrical $4^{\text {th }}$ unit, and the overlong CB line, suggest that the width of the keyboard was derived from the clavisimbalum drawing and transferred to the clavicordium.

We can now see how Strohmayer's interpretation of the clavicordium keyboard width as $93 / 7$ units was apparently confirmed: The clavicordium keyboard is, as a result of the draughting error on the CB line, about 0.5 mm wider than the $91 / 3$ units it should be. The other (and larger) part of the provenance of Stromayer's 93/7 dimension is that 3 units are identical with 7 keyheads from the bass half of the keyboard $\mathrm{h}-\mathrm{e}^{1}$, but these are the wider keys (as noted above), which leads to an inflated overall dimension, but one closer to his theoretically derived size.

53 In practice, when using dividers for a number of steps, it helps to divide up a length into smaller sections so that any marginal error in the divider is not "multiplied" by being used over the entire length. The clavisimbalum's keyboard has only 21 naturals so a different division was used, which is uniformly spaced.
54 Such a draughting aid is shown by Heyde, Musikinstrumentenbau, 1986, 58.

A further interesting detail of Arnault's draughting procedure is revealed by the keyboards of the two dulce melos instruments. Both keyboards have the compass $\mathrm{h}-\mathrm{a}^{2}$, but the widths are different and neither matches the clavisimbalum and clavicordium keyboards. However, both of these dulce melos keyboards show the same asymmetry about the $\mathrm{e}^{1} / \mathrm{f}^{1}$ junction. Closer examination with expanded photocopies and the reversed foil technique strongly suggests that the keyboards are scaled-down versions of the clavicordium keyboard, complete with its asymmetry and different spacing in each half. We may thus infer that Arnault used proportional reduction from the existing clavicordium keyboard. The clavisimbalum keyboard, by contrast, is evenly and accurately divided.

What we have arrived at is a probable order of work starting with the fimbrium, from which the clavicordium was developed, then the clavisimbalum which was made with 12 units equivalent to the 14 units of the clavicordium case length. Then a slightly overlong 8-unit CB was produced, from which the clavicordium keyboard was taken.

There are three indications that the clavisimbalum might originally have been intended to be only 12 units long. Firstly, we have the difference in size of the last unit added to make the $13^{\text {th }}$ unit, therefore possibly not made at the same time as the 12 divisions. Secondly, we have the line EF drawn across the bass end of the instrument, as if it were the tail itself. Thirdly, and most compelling, is that the bentside has been drawn with a compass only as far as F in the bass. The fact that the line of the bentside from $F$ to the bass end was clearly drawn separately from the remaining bentside, and possibly freehand, seems to show that the last unit, and this section in the bass, were an afterthought (fig. 17).

Although these considerations of the original clavisimbalum case length are somewhat speculative, what is also at issue is the ratio intended between keyboard width and case length. Clearly if the clavisimbalum was originally intended to have an 8:12 ratio (= $2: 3$ ), then both instruments were designed with this guiding principle for the width of the keyboard in relation to the case length.

Although an original 12 unit length of the clavisimbalum is not certain, we can be sure that the width of the keyboards is identical and with the same probability infer that 12 units of the clavisimbalum were intended to be identical with 14 units of the clavicordium. This reading of the drawings shows that the discrepancy of the clavicordium's keyboard with the instructions for making it is the result of an intention to use a keyboard width ( $91 / 3$ units) equal to $2 / 3$ of the case length, rather than follow the instructions, which would have led to a width of 9.1 units. One might therefore


Fig. 17: An enlarged view of the tail of the clavisimbalum. © Georges Le Cerf and Edmond René Labande: Les Traités d'Henri-Arnaut de Zwolle et de Divers Anonymes, Paris 1932 (reprint with comments by François Lesure, Internationale Gesellschaft für Musikwissenschaft, Documenta Musica, 2 ${ }^{\text {nd }}$ series, IV, Kassel 1972), plate VI
suspect that Arnault ignored the instructions because they were not his own, but taken from an earlier source.

## Conclusion

Arnault's design exhibits characteristics of the keyboard layout found in existing clavichords so he was informed of contemporary practice and perhaps not the originator. A hypothetical error in the execution of the keyboard drawing was initially thought to account for the discrepancy between the clavicordium's instructions for designing the keyboard and the drawing. Although this hypothesis covers the readily observable facts, further examination of the drawing (in part III) found, that it does not explain nuances of the keyboard division.

The clavicordium's keyboard drawing was found to be asymmetrical about the centre with a different (but even) spacing in each half. This asymmetry probably resulted from transferring a small draughting error in the clavisimbalum, and was also applied to the dulce melos' keyboards at a smaller scale.

Published facsimiles of the drawings, as Koster's measurements show, are consistently too large (c. $2 \%$ ) and the binding of the manuscript at the clavicordium drawing has shortened the first unit. After taking account of these errors, 14 units of the clavicordium were found to equal 12 units of the clavisimbalum with an accuracy confirmed by the draughting standards of the drawings. The scale ratio of the instruments was therefore 7:6 (1.6666 ${ }^{\text {r }}$ ) and the clavicordium's keyboard is $2 / 3$ of its case length.

There are indications that the clavisimbalum drawing was originally intended to be only 12 units long, which implies that both instruments had a keyboard width $2 / 3$ of the case length.

Strohmayer's interpretation of a common GI and IK length in the clavicordium and clavisimbalum required the re-defining of GI in the clavicordium in order to create the unifying feature. The length IK in the clavisimbalum, on which the hypothetically common design also depends, has no function in the drawing. Thus, Strohmayer's hypothesis is self-consistent but artificial. His interpretation required that the clavicordium keyboard have a width of $93 / 7$ units, but analysis of the drawing implied an intended size of $91 / 3$ units ( $2 / 3$ of its case length), thus, Strohmayer's size relationship 93/7:8 (1.1786) between the two instruments is incorrect.

It is concluded that the discrepancy between the clavicordium's drawing and the instructions for making the instrument is not explained by an error in the layout, nor by Strohmayer's hypothesis, but is the result of Arnault's ignoring the instructions (which he might have taken from an earlier source) and using a keyboard width $2 / 3$ of the case length.


[^0]:    In order to obtain the keyboard division construct a line descending at a right angle [i.e. $90^{\circ}$ to the line of the strings] from the middle of the $f$ key and also a line descending from the middle of the key $f^{\prime \prime}$ with two points [ $\left.f^{2}\right]$ this to [be] call[ed] line GH, which should be divided into 14 equal parts, then half a part be put from $G$ towards H [labelled I on the drawing] so that the key[head] be in the middle and from the other side [be put] the same distance [i.e half a part] in the same manner from the point H ; the total space between I and K will be 13 whole parts.

[^1]:    6 See my article, Arnaut's clavisimbalum mechanisms, in: FoMRHI Quarterly, no. 100, ed. by Jeremy Montagu, Oxford 2000, 26-33.
    7 Le Cerf, Labande, Les Traités d'Henri-Arnaut de Zwolle, 1972, 18 note 1.

[^2]:    8
    It was not known at the time of writing what size would be adopted for this reproduction. The reader wishing to reproduce the manuscript size should ensure that 13 units (the units 1-14) $=184.5$ mm . For further discussion of the size see part III.

[^3]:    22 Since the clavicordium as printed (plan 2 in Strohmayer, Traditional design, 2002, part I) has been reduced to $82 \%$ of the MS size and the clavisimbalum (plan 1b) reduced to $69 \%$ of its MS size, this discrepancy is not easily visible. In addition to this, there is no clear point mark for the position of I, which allows for vagueness, until the point has been established by following Arnault's described geometrical method. The label for i placed on the drawing is incorrectly positioned; it should be on the centre of the bridge.

[^4]:    the reader can understand what is being claimed. This is a significant difficulty in the presentation, but it is not necessary to examine this further here since the acceptance of Strohmayer's hypothesis depends more on other factors.
    37 Strohmayer, Traditional design, 2002, part III, fig. 15, 16, and 21.

[^5]:    in two stages: length HN on the fimbrium was marked out in several steps with dividers from the initial MN length. HN was then divided into 11 parts for the h string; a 3 further parts (from H/11) completed the case length of 14 units, 1 at the bass end and, 2 in the treble.
    49 This also leads to difficulties in measuring the size of keyheads if one starts at the beginning of the first unit! My comparison in fig. 14 was made from units $7-14$, thereby avoiding the shortened first unit.
    50 As far as I am aware the 14:12 relationship between the two instruments has not previously been published, but I doubt that I am the first to have noticed this.
    51 In his lectures in Vienna, as reported by Huber, Baugrößen, 1990, note 19. Strohmayer also incorporated this detail in his later article Mathematische Gestaltungsprinzipien, 2001, 41. I have described some extant instruments with this style of design in Pythagoras and the Scale Design of Early Harpsichords in France, Germany, and Italy, British Harpsichord Society (2004), published online at www.harpsichord. org.uk.

