Pythagoras and the Scale Design of Early Harpsichords in France, Germany, and Italy. **Denzil Wraight**

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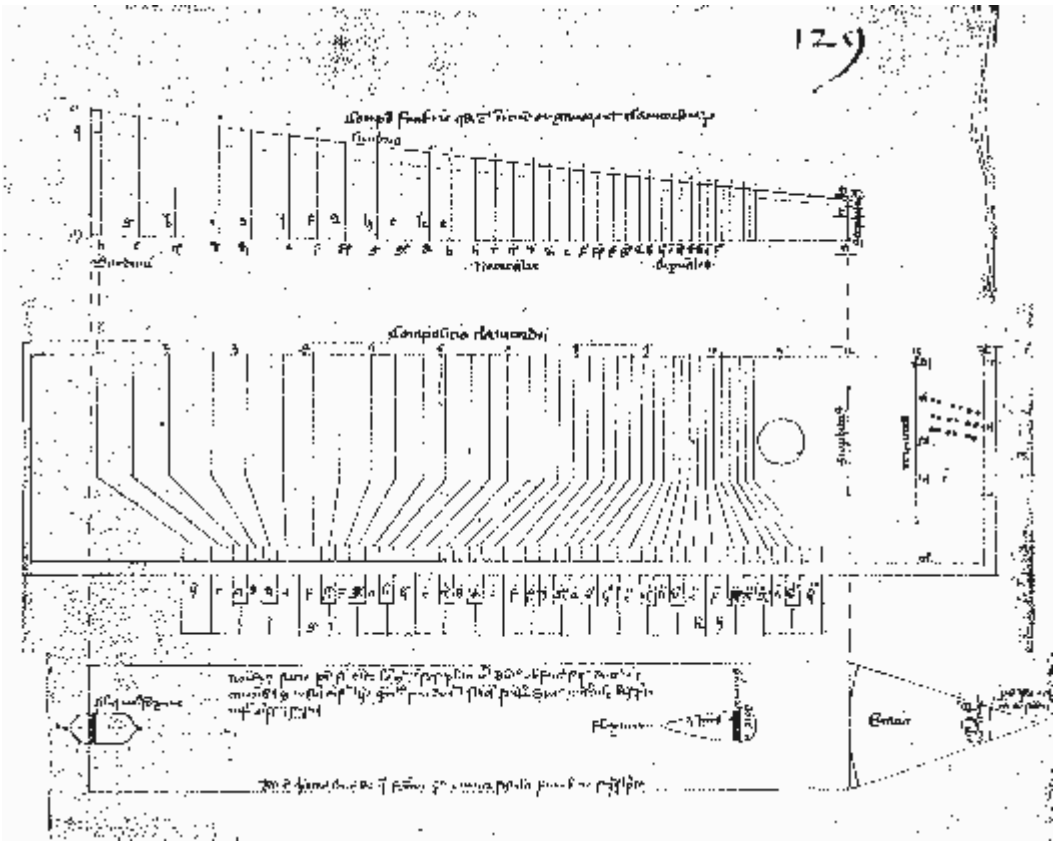
It is a custom in the physical sciences to honour the contribution of some worker in the field by naming units of measurement after him. Thus, in the measurement of the frequency of wave motion, of what used to be called "cycles per second" we have the Hertz after the German physicist Heinrich Hertz. There is no unit called the "Pythagoras", but organologists have for some time referred to a certain type of string scale design as "Pythagorean", after the ancient Greek philospher who is credited with describing the numerical relationships of intervals. When the sounding string length on a musical instrument is exactly doubled or halved at each lower or higher octave respectively, then the scale design is said to be "Pythagorean". This article investigates the use of Pythagorean scaling design in some early string keyboard instruments and shows that it was used in Italy in a way which has not been recognised until recently.

That Pythagoras' contribution of the numerical relationships between intervals also provided the basis for the physical length of strings is well enough known not to require further elaboration here. There comes a point though in the design of practical musical instruments from around 1500 when the size of the compass used effectively prevented the employment of a Pythagorean scale design throughout the compass. In other words, it became necessary to make some strings in the bass of the instrument shorter that their theoretical (Pythagorean) length.

A paradigm of Pythagorean scale design is the small fretted clavichord, such as that depicted by Arnaut de Zwolle in his manuscript of c.1440. It is shown below with the fimbrium, clavichord, and pipe aligned with hatched lines (they are displaced in the manuscript).



All the strings in this instrument have the correct theoretical length dictated by their intervallic relationships to other notes. The derivation of the clavichord fretting is clearly shown to come from the fimbrium, or scale design, which is drawn on the same sheet just above the clavichord. There is also a pipe drawn, which has the same body length as the longest string of the clavichord and the fimbrium. Thus, a relationship is illustrated where string length and pipe length are shown as identical.

The oldest surviving string keyboard instrument is an upright harpsichord, what we would now refer to as a clavicytherium, kept in the Royal College of Music museum, dated to c.1480 and thought to have been made in or near Ulm in Southern Germany. It is clear that the scale design of the speaking lengths of the strings as given below is not Pythagorean. The theoretical (Pythagorean) lengths derived from f² have been given in brackets in order to indicate the discrepancy between a Pythagorean scaling and what we find.

f²	176 mm	
f¹	310 mm	(352)
f	578 mm	(704)
F	964 mm	(1408)

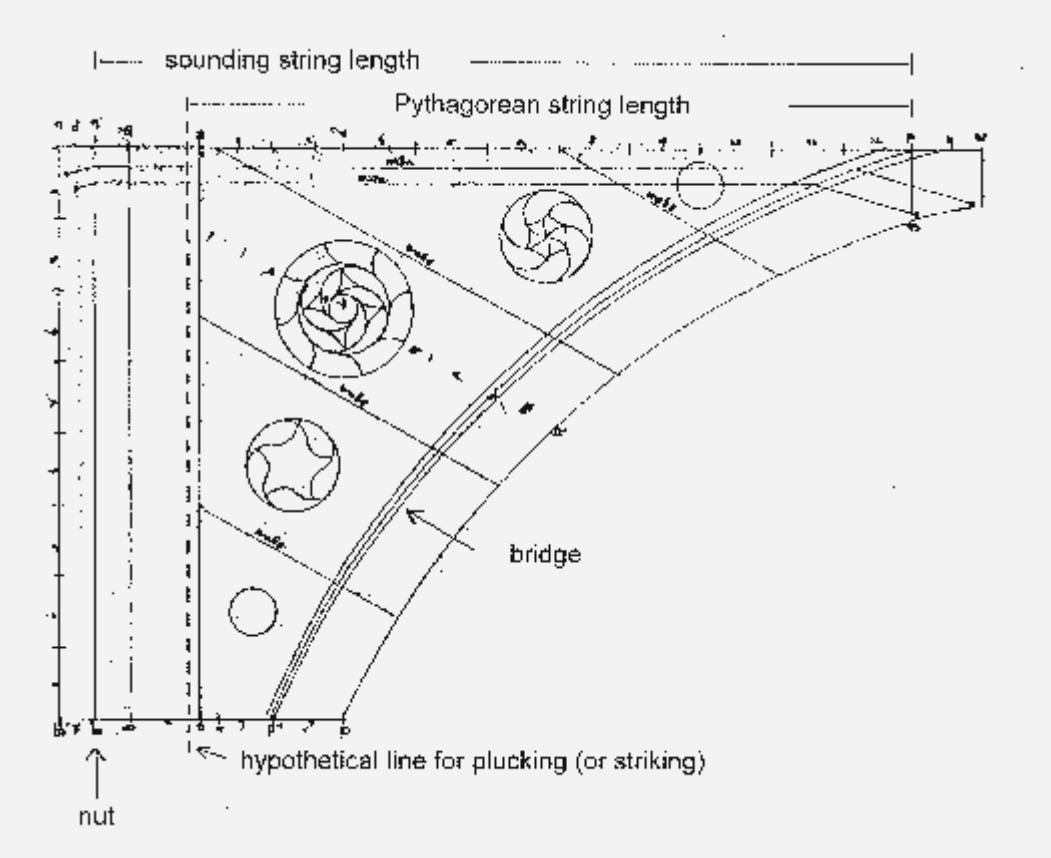
Some years ago Peter Kukelka observed in his lectures in Vienna that this instrument could be seen to have an essential similarity to the design of Arnaut's *clavisimbalum*, a wing-shaped harpsichord also depicted in the Arnaut manuscript.[1] The feature

[1] Kukelka's contribution was reported by Alfons Huber in 'Baugrößen von Saitenklavieren im 15. Jahrhundert', Das Musikinstrument 39 (Feb-March 1990), pp. 174-186, which is a significant article on this subject of early instruments, particularly on Arnaut's clavichord.

which unites the two instruments is the fact that the lengths of the strings from the bridge to the front edge of the clavicytherium soundboard, or the bridge to a nominal plucking point in the Arnaut *clavisimbalum* yield Pythagorean string lengths. In the clavicytherium the f string lengths now appear thus (after correction so that the strings are measured parallel to the spine):

f²	119 mm (238)
f¹	232 mm (232)
f	479 mm (239.5)
F	870 mm (217.5)

The figures in brackets are the scale relative to f² (i.e. 479 mm divided by 2 yields 239.5 mm). It is clear that at F the string length is too short, a compromise brought about by the size of the instrument, but the rest of the scaling is so close to a Pythagorean scale that I think we may be sure the maker laid out the position of the bridge from the front edge of the soundboard using these values. Of course he would not have used mm, and which units he used (if any) will not be investigated further here. Thus, we have a Pythagorean scale design incorporated in the instrument, but not for the *sounding* lengths of the strings. This may seem rather curious to us now some 500 years after this design was produced, but it was obviously a well-thought out part of the plan.



When we examine the *clavisimbalum* in more detail we can understand the probable thinking behind this approach. The instrument from Arnaut's manuscript has been

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Herbert Heyde's observation about Arnaut's *clavisimbalum* was published in Musikinstrumentenbau (Wiesbaden 1986), p. 160.

depicted above with some explanatory labels. The Pythagorean-scaled section of the string length is shown to the hatched, hypothetical plucking point line. When we compare these Pythagorean lengths on the *clavisimbalum* between bridge and plucking point with the fimbrium (or clavichord) we find that they are essentially the same.[2] Thus it seems that the origin of the string lengths for the *clavisimbalum* drawing is *also* to be found in the *fimbrium* and the *fimbrium* as we have seen is the prototype for a clavichord string or an organ pipe. King John of Aragon in a letter of 1388 writes of "...an instrument resembling an organ that sounds with strings" and this description, as John Koster has observed, would well fit the upright harpsichord since not only is the appearance of a small table positive similar, but the design procedure for the strings also follows that for designing organ pipes.[3]

To find the *clavisimbalum* nut position, the next step in the design is apparently to have added a length of string equal to the bridge-plucking point length, an additional length which is constant for all notes. In the clavicytherium design the situation is slightly different since the length of additional string at the first note in the treble is only *half* the length of the soundboard edge-bridge length. Thus, we can say that other factors being unchanged, Arnaut's *clavisimbalum* is an octave lower than its starting point but the clavicytherium is only a fifth lower.

This interpretation of the scale design with Pythagorean string lengths between the bridge and plucking points can also be found to apply to an Italian harpsichord (Royal College of Music [abbreviated as RCM] no. 175) as I have shown in another article.[4] This instrument is of south Italian origin, possibly from Naples, although based on the units of measurement used I consider that it may have been made in Sicily. It is now dated in the current catalogue as c.1610, but after considering the scale design and possible original compass of F,G-f^3 we might conclude that it was made much earlier, possibly as much as 100 years earlier. In this harpsichord the Pythagorean string lengths are found between the nominal plucking points and a hypothetical bridge line (the original bridge has not survived) parallel to the bentside. Again, as with the RCM clavicytherium the additional length of string in the treble is half of the Pythagorean string length, so this instrument is also nominally a fifth lower than the originating scale.

There is a surviving clavichord of early 16th-century date (no. 3, Musikinstrumenten-Museum, University of Leipzig) which can be considered to exemplify the originating scale for the RCM no. 175 harpsichord and current scholarship considers that this was probably made in Naples. As Herbert Heyde has already shown, this clavichord design is remarkably close to that of the small clavichord depicted by Arnaut in his manuscript.[5] Thus, we have an interesting example of early Italian instrument making which demonstrates the principles of harpsichord scale design as can be traced back

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[2] This subject has also recently been treated by John Koster in 'Toward a History of the Earliest Harpsichords' in Das Österreichische Cembalo, ed. Huber, A. (Tutzing 2001), pp. 17-33. An article of mine in the same volume, 'Überlegungen zu Mechanik und Mensurentwicklung im Cembalobau des 15. Jahrhunderts', pp. 79-88 formed the basis for this article.

[3] Koster, op. cit. p. 26-27.

[4] Denzil Wraight, 'The design of an early Italian harpsichord at the RCM', FoMRHI Quarterly no. 100 (2000), pp. 20-25. The RCM catalogue entry on this instrument (Keyboard Instruments, 2000, pp. 36-39) mistakenly cites another article of mine in the same journal on Arnaut's keyboard mechanisms as referring to this harpsichord.

[5] Heyde, op. cit. pp. 145-147

to Arnaut of Zwolle and his manuscript written in Burgundy. When we reflect that the clavicytherium also exhibits a similar scale design and that this instrument may have been made in or near Ulm, then we have convincing evidence that the early harpsichord designs were based on principles which knew no national boundaries.

After a little searching I found two other Italian harpsichords which may also have been designed using Pythagorean string lengths between the bridge and plucking points. Although further examination is required to confirm this hypothesis, the bentside shape which is generated by Pythagorean scaling from the plucking points is quite different from Pythagorean strings starting at the nut. Thus, such unusual instruments are relatively easy to recognise and distinguish from conventional designs. Both of these instruments appear to be from Southern Italy and probably from Naples. One is in the Museum of Fine Arts, Boston, no. 1986.518, the other in Castello Sforzesco, Milan, no. 447 (previously Gallini no. 579).

This shows that the supposedly *Italian* tradition of Pythagorean scaling, as exemplified by 16th-century Venetian harpsichords, where the *speaking* string lengths are Pythagorean to about c in the tenor was only *one* tradition. What happened in Venice c.1500 we do not presently know, and are unlikely to discover since the earliest known Venetian harpsichord was built in 1530 by Alessandro Trasuntino (private ownership, Siena, Italy) and an earlier one will probably not be discovered.

In describing the Arnaut *clavisimbalum* strings as Pythagorean between a nominal plucking point and the bridge I gave emphasis to an observation made earlier by Kukelka and Heyde. Given that the RCM clavicytherium was designed in this way, the observation about Arnaut's design *appears* to be justified. However, it is *not* the method that Arnaut describes for building his *clavisimbalum*. His approach is to fix the proportions of the width and length of the instrument, then find a point outside the instrument in order to draw the bentside and the bridge as arcs of a circle. It is geometrically impossible for the arc of a circle to describe the same line as the exponential curve generated by Pythagorean scaling from the nominal plucking point line (or edge of the soundboard). Thus, we have a tension between two different methods and this takes us to the heart of another problem concerning Arnaut's designs: were they his own creations or did they reflect actual instrument making practice? In other words was Arnaut reporting designs he had invented or was he producing his own theoretical explanations for how practising instrument makers went about their work? If the latter, then his method would fit approximately with the result of what instrument makers produced yet not reproduce their method.

Current scholarship has not produced a definitive answer to these questions. Wolfgang Strohmayer sees Arnaut as the inventor of both designs and gives a complicated yet completely consistent interpretation which links the clavichord and *clavisimbalum* as generated by similar processes of modular construction.[6] The matter is too intricate to be discussed further here.

Arnaut reports on a number of instruments in his manuscript. Since he refers to organs, lutes, a dulce melos, and four different types of action which can be used in a

[6] Strohmayer's ideas have been published in German, but will be more accessible to many readers in an English-language article 'Traditional Design Principles in the Early History of Keyboard Instruments', Clavichord International, Parts 1 & 2 May 2002, pp. 13-21, Part 3, November 2002, pp. 44-46.

clavisimbalum this appears to be prima facie evidence that he was *reporting* on instrument making, and might not have made all these instruments himself. Undoubtedly he was well informed but he may not have been informed about every detail of the design process.

The unifying idea behind the instrument design of the RCM clavicytherium and the RCM no. 175 Italian harpsichord is that the bentside shape is generated *from* the strings and not vice versa. In other words one starts with the strings and creates a case which holds them rather than making a case and fitting the strings in. This design process is that which we see in Arnaut's clavichord and is what we might also call Pythagorean in its spirit. The strings are not only the physical elements which when set in motion are the immediate causation of the sound, but they are also the basis of the instrument design.

This point may seem so elementary as to require little exposition but in fact the implications of this approach are far reaching since some recent contributions in the organological literature at explaining the creative procedure in harpsichord making work from the assumption that one *starts* with the case dimensions.[7] This interpretation may be correct for some designs, but as I will show in another publication, some Venetian harpsichord makers did not even measure the length of the instrument; that was derived indirectly from the strings; the strings were the basis of the design.

I started this article with reference to Pythagoras and the reader will see that there is a thread running through occidental instrument making which derives its entire logic from the principles of intervals being represented by proportionally-derived string lengths. It has not been known until recently that Italian harpsichords also used this early form of non-Pythagorean *sounding* string lengths, but which on further examination can be seen to have had a Pythagorean basis. This evidence strengthens the case for a tradition of instrument design which used Pythagorean string lengths as the *basis* of the scale design, even when the sounding lengths were non-Pythagorean. Besides amplifying our view of early scaling practice, this evidence will contribute to our interpretation of Arnaut's manuscript and its place as a document on instrument making practice.

[7] Two substantial contributions in this area are from Grant O'Brien, 'The use of simple geometry and the local unit of measurement in the design of Italian stringed keyboard instruments: an aid to attribution and to organological analysis, Galpin Society Journal 52 (1999), pp. 108-171, and Stephen Birkett & William Jurgenson, 'Geometrical Methods in Stringed Keyboard Instrument Design', Galpin Society Journal 54 (2001), pp. 242-284.